**Electric Susceptibility**

We’ll take a look at the dynamic electric susceptibility. Basically same considerations as in the time-independent case. Only now, the system won’t be able, presumably, to settle into thermodynamic equilibrium. We’ll have to use non-equilibrium statistical mechanics to calculate these response functions.

**Electric Susceptibility Definitions**

So the general conceit of the electric susceptibility is that we have external charges, whose net external electric field induces an induced charge in the metal. But to examine it a little further, the induced charges will then create their own net induced electric field, which will combine with the net external electric field, and this total field is what will have the final say on the positions/movements of the induced charges. But in a Free Electrons folder, we can’t have electrons interacting with each other (except maybe via the Pauli Exclusion Principle). So they can’t respond to each other’s fields. And since electrons really respond to the *total* (induced plus external) field present, and since they are only allowed to respond to the external field, this means that they can’t really even create fields themselves.

So in this case, we’ll have an electric susceptibility χirr(q,ω) = something. But there will be no distinction (below) between φ and φf (φf = potential coming from external charges), the potential coming from all charges vs. the potential coming from the test charges, because the induced charges, whose response we want to calculate, cannot create a field of their own if they don’t interact with each other. So then ε(q) would be the same as ε0 (or, 1, in our faux Gaussian units). And then χ(q,ω) would have to equal χirr(q,ω). So we’d no longer have the equation ε(q,ω) = 1 – v(q)χirr(q,ω). This is necessary, because we can/will calculate χirr(q,ω) in this folder. And we’ll see it has an imaginary part. But then this would imply the absorptivity A(q,ω)=ωImεeff(q,ω) is non-zero. But this would mean that photons are being absorbed by electrons (manifested by plasmons) even while there is no place (plasmons) for the energy to go.

So with this in mind, I guess we can go ahead and calculate χirr(q,ω), and simultaneously stipulate ε(q,ω) = 1. And this means that we’ll be calculating the induced charges response to the external charges, but nothing else, so far. That will be sufficient to calculate the ‘free’ electron dielectric suseptibility. In the e-e interaction folder, we’ll look at corrections to this result.

For review, I’m going to reproduce the discussion in the EM folder/Metals file. So the conceit is that we introduce a free charge/current density, which gives rise to free fields, and ultimately induces a responsive charge/current density. I’ll redo the analysis we did in EM file for metals, because of the slightly different pre-factors we’re using, mainly due to fact we’re in fake ‘Gaussian’ units now [basically can go from SI to fake Gaussian by ε0 → 1/4π, μ0 → 4π]. Referring to the Units file, if we were to take the electric susceptibility equation:



and convert to fake Gaussian units, we’d get:



But instead, we’re going to write:



So this χirr is smaller than the χirr we defined in the EM folder by a factor of 4π, and the ‘advantage’ of doing so is it allows us to put things more easily in terms of the Coulomb potential’s Fourier transform. Anyway, this is equivalent to, taking the spatial/temporal Fourier transform of both sides:



Now how do all of these functions relate? We can use Maxwell’s equations, like was done before. We’ll use:



Taking the Fourier transform of both sides,



Plugging in our induced ρ,



And now we can define the proportionality between ρf and φ as:



where υ(q) = 4π/q2. This is called υ(q) because the Coulomb potential has the same Fourier transform, in Gaussian units, sans the factors of e. Must be careful not to confuse this υ(q) with the actual potential V(q) that *includes* the e’s. Now let’s relate the induced charge to the free potential. By definition, this is:



and can see χ(q,ω) must equal χirr(q,ω) because φ = φf. Finally, from these two highlighted equations, we can relate ρind to ρf via:



**Appendix**

Should consider converting these susceptibilities back to SI. Well, basically you have to get χirr, reconvert it to ‘true’ fake Gaussian units by multiplying by 4π, and then do the unit analysis of multiplying it by factors of (4πε0)p(μ0/4π)q(ℏ)r (since we’ll be using fake Gaussian + Natural units). Well turns out we just need to multiply χirr, *overall*, by (4πε0)-1. Can see example in Non-equilibrium folder/Lindhard Properties.